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A Note on Estimating Constant Growth Terminal Values With Inflation

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In path-breaking articles, Bradley and Jarrell develop an analysis that properly accounts for inflation in the context of constant growth valuation models. They show that many traditional applications of the constant growth model err by failing to properly account for the impact of inflation on the existing capital stock. Despite the publication of the Bradley and Jarrell papers, many leading valuation texts, including Damodaran and Koller, Goedhart, and Wessels, still employ variations of the traditional model, and a good deal of debate remains regarding the applicability of the Bradley-Jarrell approach. In that light, this short note offers a particularly simple and intuitive derivation of the Bradley-Jarrell results that makes it clear why the traditional models are in error when applied to actual or forecasted net operating profit after tax derived from GAAP-based financial statements or forecasts.

Introduction

In most corporate valuations the continuing, or terminal, value accounts for a majority of the total estimated value. In the case of smaller and more rapidly growing companies, the terminal value often exceeds 100% of the estimated value. Consequently, the manner in which the terminal value is calculated is an issue of first order importance in any valuation exercise. The most widely adopted procedure is to project cash flows year by year up to the point when the firm has reached a steady state and then to apply a constant growth model.

In path-breaking articles, Bradley and Jarrell (2008, 2011) demonstrate that traditional constant growth models incorporating the traditional plowback ratio fail to take proper account of the impact of inflation on the existing capital stock and thereby overstate required investment for growth and underestimate the terminal value. Their work set off a flurry of responses, including articles by Friedl and Schweitzler (2008), Jennergren (2011), and Kiechle and Lampenius (2012a, 21012b). Despite the debate, many leading books on valuation, including Damodaran (2012) and Koller, Goedhart, and Wessels (2015), fail to take account of inflation as recommended by Bradley and Jarrell. This may be because the Bradley and Jarrell analysis introduces new terms not used in standard accounting as part of a relatively complex algebraic analysis. For that reason, this short note presents a simple derivation of the Bradley-Jarrell effect that makes it clear how the traditional approach is in error.

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The Bradley-Jarrell Analysis and Investment Required for Growth

In the following analysis, all magnitudes with capital letters are nominal and small letters are real. Using that notation, the traditional analysis of the relation between plowback of earnings and growth before Bradley and Jarrell begins with the identity that

$$\mathrm{NOPAT}_t = \mathrm{NOPAT}_{t-1} + \Delta \mathrm{IC}_t \times \mathrm{ROIC}. \tag{1}$$

In equation (1), NOPAT is net operating profit after tax, ΔIC is the increase in the nominal capital stock, and ROIC is the return on invested capital. If it is assumed (wrongly as it turns out) that ΔIC comes solely from retained earnings, and if the plowback ratio for retained earnings, k, is defined as $\Delta \mathrm{IC}_{t-1}/\mathrm{NOPAT}_{t-1}$, then substituting for $\Delta \mathrm{IC}_t$ gives

$$\mathrm{NOPAT}_t = \mathrm{NOPAT}_{t-1} + \mathrm{k} \times \mathrm{NOPAT}_{t-1} \times \mathrm{ROIC}. \tag{2}$$

From (2) it follows that

$$\mathrm{G} = (\mathrm{NOPAT}_t - \mathrm{NOPAT}_{t-1})/\mathrm{NOPAT}_{t-1} = \mathrm{k} \times \mathrm{ROIC}. \tag{3}$$

Equation (3) is the classic relation that Damodaran and Koller, Goedhart, and Wessels, among others, use to solve for the plowback ratio, k, necessary to fund growth, G, given return on invested capital, ROIC. The problem is that in the presence of steady state inflation, equations (2) and (3) are wrong when applied to NOPAT derived from GAAP-based financial statements or forecasts.

The traditional analysis goes off track at equation (2). The added invested capital necessary to produce growth does not come solely from retained earnings, it also comes from inflationary appreciation in the existing capital stock. Defining NNI as the net new investment of fresh dollars in excess of replacement expenditures, the nominal increase in the capital stock is given by

$$\Delta IC_t = IC_t - IC_{t-1} = NNI_t + \pi \times IC_{t-1}, \quad (4)$$

where π is the steady state rate of inflation. Equation (4) demonstrates the cash that must be retained and reinvested in excess of replacement expenditures, that is, NNI, is less than the increase in nominal invested capital. The difference is the inflation term on the firm's invested capital.

In steady state, equation (4) shows that a component of the increase in the firm's invested capital is due to inflation acting on the existing capital stock. This should be distinguished from the impact of inflation on the capital expenditures required to maintain a firm's invested capital. It is possible that inflation will cause replacement capital expenditures to exceed depreciation. However, this is not necessarily the case. If the capital the firm uses, such as computer equipment, is improving at a rate greater than inflation, replacement cost will be less than depreciation. We discuss this issue further below.

The problem with equation (3), when applied to NOPAT derived from GAAP-based financial statements or forecasts, is that by ignoring the impact of inflation on the stock of invested capital, it typically overstates the required cash plowback and, thereby, understates free cash flow, FCF, and undervalues the firm.

It is important to recognize that in steady state, inflation is assumed to be constant such that all prices and financial metrics, including revenue, NOPAT, and asset and liability values, increase at the same rate year after year. Accordingly, in steady state all financial variables grow at the same rate. If they do not, the ratio of two variables with different growth rates will either diverge to infinity or converge to zero, neither of which makes any sense. Thus, if FCF is growing at the rate G, then invested capital must be growing at the same rate. Therefore, dividing equation (4) by IC_{t-1} yields

$$G = \Delta IC_t/IC_{t-1} = NNI_t/IC_{t-1} + \pi. \quad (5)$$

To tie equation (4) to the Bradley and Jarrell analysis it is necessary to introduce the concept of net cash flow, NCF. The new term is required because it is generally not the case that accounting depreciation equals economic depreciation. As a result, when NOPAT and depreciation are derived from GAAP-based financial statements or forecasts, the net cash flow before plowback is given by the relation[1]

$$\begin{aligned} NCF = {} & NOPAT + Depreciation \\ & - \text{Replacement capital expenditures.} \end{aligned} \quad (6)$$

It is NCF that is the actual cash return on the capital stock because of the need to cover replacement capital expenditures, not NOPAT. It is appropriate, therefore, to measure return on investment and the cash-based plowback ratio relative to NCF, not NOPAT. Therefore, the Bradley-Jarrell cash-based plowback ratio is given by

$$k' = \text{Net new cash investment(NNI)}/NCF, \quad (7)$$

The prime is used to emphasize that the plowback ratio is now defined with respect to NCF. Similarly, the real return on invested capital, r, is also defined with respect to NCF,

$$r = NCF_t/[(1+\pi) \times (IC_{t-1})]. \quad (8)$$

Rearranging equation (8),

$$IC_{t-1} = NCF_t/[(1+\pi) \times r]. \quad (9)$$

Substituting (9) into (5) and using the definition of k',

$$G = k' \times r \times (1+\pi) + \pi. \quad (10)$$

Equation (10) is the Bradley-Jarrell relation between growth return on investment and plowback. To see that more directly, add and subtract $k' \times \pi$ on the right-hand side and collect terms to give

$$G = k' \times (r + \pi + \pi \times r) + (1 - k') \times \pi.$$

Recognizing from the Fisher relationship that $(r + \pi + \pi \times r) = R$, the nominal return on invested capital the equation reduces to

$$G = k' \times R + (1 - k') \times \pi. \quad (11)$$

This is the equation derived by Bradley and Jarrell. Solving equation (11) for k' gives the fraction of NCF that must be plowed back in addition to the inflationary expansion of the capital stock to fund growth at a rate G.

Errors related to the failure to take account of the impact of inflation on the nominal capital stock are not confined to the basic analysis given by equations (2)–(3). In his posted valuations, Damodaran uses the sales-to-capital ratio to calculate what he calls required investment.[2] In this context, the required investment equals the change in revenue from one period to the next divided by the sales-to-capital ratio. The required investment calculated in this fashion is then deducted when computing free cash flow. But that is the wrong number to deduct. The required investment is exactly analogous to ΔIC. It is the *total* increase in nominal invested capital required to maintain the relation between invested capital and

[1] For simplicity, the change in working capital is assumed to be incorporated as part of the plowback investment.

[2] See http://aswathdamodaran.blogspot.com/.

revenue. It is *not* the amount of fresh cash that must be plowed back and, therefore, *not* the amount that should be deducted when calculating free cash flow. Instead the amount that should be deducted is ΔIC (the required increase in the nominal capital stock) minus the inflationary increase in the existing capital stock in excess of replacement capital expenditures. This fact becomes clear in the limit when growth is due solely to inflation. Revenue continues to grow at the rate of inflation, so Damodaran's calculation still leads to positive required investment. However, that required investment is entirely accounted for by the inflationary appreciation of the capital stock. There is no need for any plowback of fresh cash, and, therefore, no deduction is appropriate when computing free cash flow.

The bottom line is this. The equations that tie growth and reinvestment in a constant growth model actually tie growth to the increase in the nominal capital stock. As Bradley and Jarrell stress, the increase in the nominal capital stock comes from two sources: plowback of fresh dollars and the inflationary expansion of the preexisting capital stock. Replacement capital expenditures are required to maintain the preexisting capital stock; however, it is only the first term, the plowback of fresh dollars, that should be deducted from net cash flow when computing free cash flow and calculating the terminal value. The traditional model still employed in many textbooks to describe the relation between growth and investment overlooks this fact and attributes the entire increase in the nominal capital stock to the plowback of retained earnings. As such, when applied to NOPAT derived from GAAP-based financial statements or forecasts, it typically understates free cash flow and, thereby, terminal value, often by a significant amount.

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